1 NUMBER

Karl Friedrich Gauss (1777–1855) was the son of a German labourer and is thought by many to have been the greatest all-round mathematician of all time. He considered that his finest discovery was the method for constructing a regular seventeen-sided polygon. This was not of the slightest use outside the world of mathematics, but was a great achievement of the human mind. Gauss would not have understood the modern view held by many that mathematics must somehow be 'useful' to be worthy of study.

1	Identify and use natural numbers, integers, prime numbers, square numbers, rational and irrational numbers; continue a given number sequence; recognise patterns in sequences and generalise to simple algebraic statements
6	Use the standard form $A \times 10^n$
7	Use the four rules for calculations with whole numbers, decimal fractions and vulgar fractions
8	Make estimates, give approximations and round off answers to reasonable accuracy
9	Obtain appropriate upper and lower bounds to solutions of simple problems
10	Demonstrate an understanding of ratio, direct and inverse proportion and common measures of rate; divide a quantity in a given ratio; use scales in practical situations; calculate average speed
11	Calculate percentage increase or decrease; carry out calculations involving reverse percentages
12	Use an electronic calculator efficiently
15	Calculate using money and convert from currency to another
16	Solve problems on simple interest and compound interest

1.1 Arithmetic

Decimals

Example

Evaluate: (a) $7{\cdot}6 + 19$ (b) $3{\cdot}4 - 0{\cdot}24$ (c) $7{\cdot}2 \times 0{\cdot}21$

(d) $0{\cdot}84 \div 0{\cdot}2$ (e) $3{\cdot}6 \div 0{\cdot}004$

(a) $\begin{array}{r} 7{\cdot}6 \\ +\,19{\cdot}0 \\ \hline 26{\cdot}6 \end{array}$ (b) $\begin{array}{r} 3{\cdot}40 \\ -\,0{\cdot}24 \\ \hline 3{\cdot}16 \end{array}$ (c) $\begin{array}{r} 7{\cdot}2 \\ \times\,0{\cdot}21 \\ \hline 72 \\ 1440 \\ \hline 1{\cdot}512 \end{array}$

No decimal points in the working, '3 figures after the points in the question *and* in the answer'.

(d) $0{\cdot}84 \div 0{\cdot}2 = 8{\cdot}4 \div 2$

$$2\overline{)8{\cdot}4}\quad 4{\cdot}2$$

Multiply both numbers by 10 so that we can divide by a whole number.

(e) $3{\cdot}6 \div 0{\cdot}004 = 3600 \div 4$
$= 900$

Exercise 1

Evaluate the following without a calculator:

1. $7{\cdot}6 + 0{\cdot}31$	**2.** $15 + 7{\cdot}22$	**3.** $7{\cdot}004 + 0{\cdot}368$	**4.** $0{\cdot}06 + 0{\cdot}006$
5. $4{\cdot}2 + 42 + 420$	**6.** $3{\cdot}84 - 2{\cdot}62$	**7.** $11{\cdot}4 - 9{\cdot}73$	**8.** $4{\cdot}61 - 3$
9. $17 - 0.37$	**10.** $8{\cdot}7 + 19{\cdot}2 - 3{\cdot}8$	**11.** $25 - 7{\cdot}8 + 9{\cdot}5$	**12.** $3{\cdot}6 - 8{\cdot}74 + 9$
13. $20{\cdot}4 - 20{\cdot}399$	**14.** $2{\cdot}6 \times 0{\cdot}6$	**15.** $0{\cdot}72 \times 0{\cdot}04$	**16.** $27{\cdot}2 \times 0{\cdot}08$
17. $0{\cdot}1 \times 0{\cdot}2$	**18.** $(0{\cdot}01)^2$	**19.** $2{\cdot}1 \times 3{\cdot}6$	**20.** $2{\cdot}31 \times 0{\cdot}34$
21. $0{\cdot}36 \times 1000$	**22.** $0{\cdot}34 \times 100\,000$	**23.** $3{\cdot}6 \div 0{\cdot}2$	**24.** $0{\cdot}592 \div 0{\cdot}8$
25. $0{\cdot}1404 \div 0{\cdot}06$	**26.** $3{\cdot}24 \div 0{\cdot}002$	**27.** $0{\cdot}968 \div 0{\cdot}11$	**28.** $600 \div 0{\cdot}5$
29. $0{\cdot}007 \div 4$	**30.** $2640 \div 200$	**31.** $1100 \div 5{\cdot}5$	**32.** $(11 + 2{\cdot}4) \times 0{\cdot}06$
33. $(0{\cdot}4)^2 \div 0{\cdot}2$	**34.** $77 \div 1000$	**35.** $(0{\cdot}3)^2 \div 100$	**36.** $(0{\cdot}1)^4 \div 0{\cdot}01$
37. $\dfrac{92 \times 4{\cdot}6}{2{\cdot}3}$	**38.** $\dfrac{180 \times 4}{36}$	**39.** $\dfrac{0{\cdot}55 \times 0{\cdot}81}{4{\cdot}5}$	**40.** $\dfrac{63 \times 600 \times 0{\cdot}2}{360 \times 7}$

Exercise 2

1. A maths teacher bought 40 calculators at \$8·20 each and a number of other calculators costing \$2·95 each. In all she spent \$387. How many of the cheaper calculators did she buy?

2. At a temperature of 20°C the common amoeba reproduces by splitting in half every 24 hours. If we start with a single amoeba how many will there be after (a) 8 days, (b) 16 days?

3. Copy and complete.

$3^2 + 4^2 + 12^2 = 13^2$

$5^2 + 6^2 + 30^2 = 31^2$

$6^2 + 7^2 + \quad = $

$x^2 + \quad + \quad = $

You can find out about square numbers on page 5.

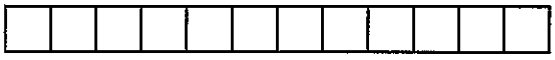
4. Find all the missing digits in these multiplications.

(a) $5* \times 9 = **6$ (b) $*7 \times * = 4*6$ (c) $5* \times * = 1*4$

5. Pages 6 and 27 are on the same (double) sheet of a newspaper. What are the page numbers on the opposite side of the sheet? How many pages are there in the newspaper altogether?

6. Use the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 once each and in their natural order to obtain an answer of 100. You may use only the operations $+$, $-$, $\times$, $\div$.

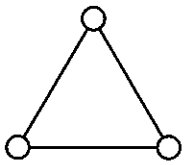
7. The ruler below has eleven marks and can be used to measure lengths from one unit to twelve units.



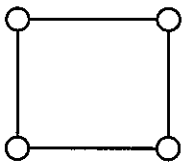
Design a ruler which can be used to measure all the lengths from one unit to twelve units but this time put the minimum possible number of marks on the ruler.

8. Each packet of washing powder carries a token and four tokens can be exchanged for a free packet. How many free packets will I receive if I buy 64 packets?

9. Put three different numbers in the circles so that when you add the numbers at the end of each line you always get a square number.



10. Put four different numbers in the circles so that when you add the numbers at the end of each line you always get a square number.



11. A group of friends share a bill for \$13·69 equally between them. How many were in the group?

Fractions

Common fractions are added or subtracted from one another directly only when they have a common denominator.

Remember
The order of operations follows the BODMAS rule:
Brackets then
powers Of then
Divide then
Multiply then
Add then
Subtract

Example

Evaluate: (a) $\frac{3}{4}+\frac{2}{5}$ (b) $2\frac{3}{8}-1\frac{5}{12}$ (c) $\frac{2}{5}\times\frac{6}{7}$ (d) $2\frac{2}{5}\div 6$

(a) $\frac{3}{4}+\frac{2}{5}=\frac{15}{20}+\frac{8}{20}$
$=\frac{23}{20}$
$=1\frac{3}{20}$

(b) $2\frac{3}{8}-1\frac{5}{12}=\frac{19}{8}-\frac{17}{12}$
$=\frac{57}{24}-\frac{34}{24}$
$=\frac{23}{24}$

(c) $\frac{2}{5}\times\frac{6}{7}=\frac{12}{35}$

(d) $2\frac{2}{5}\div 6=\frac{12}{5}\div\frac{6}{1}$
$=\frac{2}{5}\times\frac{1}{6_1}=\frac{2}{5}$

Exercise 3

Evaluate and simplify your answer.

1. $\frac{3}{4}+\frac{4}{5}$ **2.** $\frac{1}{3}+\frac{1}{8}$ **3.** $\frac{5}{6}+\frac{6}{9}$ **4.** $\frac{3}{4}-\frac{1}{3}$ **5.** $\frac{3}{5}-\frac{1}{3}$

6. $\frac{1}{2}-\frac{2}{5}$ **7.** $\frac{2}{3}\times\frac{4}{5}$ **8.** $\frac{1}{7}\times\frac{5}{6}$ **9.** $\frac{5}{8}\times\frac{12}{13}$ **10.** $\frac{1}{3}\div\frac{4}{5}$

11. $\frac{3}{4}\div\frac{1}{6}$ **12.** $\frac{5}{6}\div\frac{1}{2}$ **13.** $\frac{3}{8}+\frac{1}{5}$ **14.** $\frac{3}{8}\times\frac{1}{5}$ **15.** $\frac{3}{8}\div\frac{1}{5}$

16. $1\frac{3}{4}-\frac{2}{3}$ **17.** $1\frac{3}{4}\times\frac{2}{3}$ **18.** $1\frac{3}{4}\div\frac{2}{3}$ **19.** $3\frac{1}{2}+2\frac{3}{5}$ **20.** $3\frac{1}{2}\times 2\frac{3}{5}$

21. $3\frac{1}{2}\div 2\frac{3}{5}$ **22.** $\left(\frac{3}{4}-\frac{2}{3}\right)\div\frac{3}{4}$ **23.** $\left(\frac{3}{5}+\frac{1}{3}\right)\times\frac{5}{7}$ **24.** $\dfrac{\frac{3}{8}-\frac{1}{5}}{\frac{7}{10}-\frac{2}{3}}$ **25.** $\dfrac{\frac{2}{3}+\frac{1}{5}}{\frac{3}{4}-\frac{1}{3}}$

26. Arrange the fractions in order of size:
(a) $\frac{7}{12}, \frac{1}{2}, \frac{2}{3}$ (b) $\frac{3}{4}, \frac{2}{3}, \frac{5}{6}$ (c) $\frac{1}{3}, \frac{17}{24}, \frac{5}{8}, \frac{3}{4}$ (d) $\frac{5}{6}, \frac{8}{9}, \frac{11}{12}$

27. Find the fraction which is mid-way between the two fractions given:
(a) $\frac{2}{5}, \frac{3}{5}$ (b) $\frac{5}{8}, \frac{7}{8}$ (c) $\frac{2}{3}, \frac{3}{4}$ (d) $\frac{1}{3}, \frac{4}{9}$ (e) $\frac{4}{15}, \frac{1}{3}$ (f) $\frac{3}{8}, \frac{11}{24}$

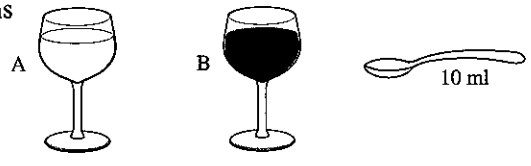
28. In the equation below all the asterisks stand for the same number. What is the number?

$$\left[\frac{*}{*}-\frac{*}{6}=\frac{*}{30}\right]$$

29. When it hatches from its egg, the shell of a certain crab is 1 cm across. When fully grown the shell is approximately 10 cm across. Each new shell is one-third bigger than the previous one. How many shells does a fully grown crab have during its life?

30. Glass A contains 100 ml of water and glass B contains 100 ml of wine.

A 10 ml spoonful of wine is taken from glass B and mixed thoroughly with the water in glass A. A 10 ml spoonful of the mixture from A is returned to B. Is there now more wine in the water or more water in the wine?



Fractions and decimals

A decimal is simply a fraction expressed in tenths, hundredths etc.

Example

Change (a) $\frac{7}{8}$ to a decimal (b) 0·35 to a fraction. (c) $\frac{1}{3}$ to a decimal.

(a) $\frac{7}{8}$, divide 8 into 7 (b) $0{\cdot}35 = \frac{35}{100} = \frac{7}{20}$ (c) $\frac{1}{3}$, divide 3 into 1

$\frac{7}{8} = 0{\cdot}875$ $8\overline{)7{\cdot}000}$ $\frac{1}{3} = 0{\cdot}\dot{3}$ (0·3 recurring) $3\overline{)1{\cdot}0^{1}0^{1}0^{1}000}$

Exercise 4

In questions **1** to **24**, change the fractions to decimals.

1. $\frac{1}{4}$	**2.** $\frac{2}{5}$	**3.** $\frac{4}{5}$	**4.** $\frac{3}{4}$	**5.** $\frac{1}{2}$	**6.** $\frac{3}{8}$
7. $\frac{9}{10}$	**8.** $\frac{5}{8}$	**9.** $\frac{5}{12}$	**10.** $\frac{1}{6}$	**11.** $\frac{2}{3}$	**12.** $\frac{5}{6}$
13. $\frac{2}{7}$	**14.** $\frac{3}{7}$	**15.** $\frac{4}{9}$	**16.** $\frac{5}{11}$	**17.** $1\frac{1}{5}$	**18.** $2\frac{5}{8}$
19. $2\frac{1}{3}$	**20.** $1\frac{7}{10}$	**21.** $2\frac{3}{16}$	**22.** $2\frac{2}{7}$	**23.** $2\frac{6}{7}$	**24.** $3\frac{19}{100}$

In questions **25** to **40**, change the decimals to fractions and simplify.

25. 0·2	**26.** 0·7	**27.** 0·25	**28.** 0·45
29. 0·36	**30.** 0·52	**31.** 0·125	**32.** 0·625
33. 0·84	**34.** 2·35	**35.** 3·95	**36.** 1·05
37. 3·2	**38.** 0·27	**39.** 0·007	**40.** 0·000 11

Evaluate, giving the answer to 2 decimal places:

41. $\frac{1}{4} + \frac{1}{3}$	**42.** $\frac{2}{3} + 0{\cdot}75$	**43.** $\frac{8}{9} - 0{\cdot}24$	**44.** $\frac{7}{8} + \frac{5}{9} + \frac{2}{11}$
45. $\frac{1}{3} \times 0{\cdot}2$	**46.** $\frac{5}{8} \times \frac{1}{4}$	**47.** $\frac{8}{11} \div 0{\cdot}2$	**48.** $\left(\frac{4}{7} - \frac{1}{3}\right) \div 0{\cdot}4$

Arrange the numbers in order of size (smallest first)

49. $\frac{1}{3}$, 0·33, $\frac{4}{15}$	**50.** $\frac{2}{7}$, 0·3, $\frac{4}{9}$	51. 0·71, $\frac{7}{11}$, 0·705	**52.** $\frac{4}{13}$, 0·3, $\frac{5}{18}$

1.2 Number facts and sequences

Number facts

- An *integer* is a whole number. e.g. 2, −3 ...
- A *prime* number is divisible only by itself and by one. e.g. 2, 3, 5, 7, 11, 13 ...
- The *multiples* of 12 are 12, 24, 36, 48 ...
- The *factors* of 12 are 1, 2, 3, 4, 6, 12.
- A *square number* is the result of multiplying a number by itself. e.g. $5 \times 5 = 25$ so 25 is a square number.
- A *cube number* is the result of multiplying a number by itself three times. e.g. $5 \times 5 \times 5 = 125$, so 125 is a cube number.

Exercise 5

1. Which of the following are prime numbers?
   3, 11, 15, 19, 21, 23, 27, 29, 31, 37, 39, 47, 51, 59, 61, 67, 72, 73, 87, 99
2. Write down the first five multiples of the following numbers:
   (a) 4 (b) 6 (c) 10 (d) 11 (e) 20
3. Write down the first six multiples of 4 and of 6. What are the first two *common* multiples of 4 and 6? [i.e. multiples of both 4 and 6]
4. Write down the first six multiples of 3 and of 5. What is the lowest common multiple of 3 and 5?
5. Write down all the factors of the following:
   (a) 6 (b) 9 (c) 10 (d) 15 (e) 24 (f) 32
6. (a) Is 263 a prime number?
   By how many numbers do you need to divide 263 so that you can find out?
   (b) Is 527 a prime number?
   (c) Suppose you used a computer to find out if 1147 was a prime number. Which numbers would you tell the computer to divide by?
7. Make six prime numbers using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 once each.

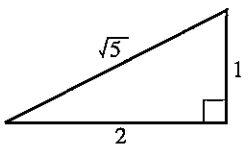
Rational and irrational numbers

- A rational number can always be written exactly in the form $\frac{a}{b}$ where a and b are whole numbers.

$\frac{3}{7}$	$1\frac{1}{2} = \frac{3}{2}$	$5{\cdot}14 = \frac{257}{50}$	$0{\cdot}\dot{6} = \frac{2}{3}$
All these are rational numbers.			

- An irrational number cannot be written in the form $\frac{a}{b}$.
  $\sqrt{2}$, $\sqrt{5}$, π, $\sqrt[3]{2}$ are all irrational numbers.
- In general $\sqrt{n}$ is irrational unless n is a square number.

In this triangle the length of the hypotenuse is *exactly* $\sqrt{5}$.
On a calculator, $\sqrt{5} = 2{\cdot}236068$. This value of $\sqrt{5}$ is *not* exact and is correct to only 6 decimal places.



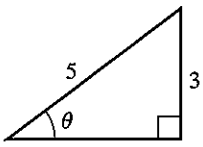
Exercise 6

1. Which of the following numbers are rational?

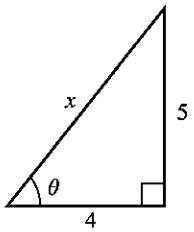
$\frac{\pi}{2}$	$\sqrt{5}$	$(\sqrt{17})^2$	$\sqrt{3}$
$3{\cdot}14$	$\frac{\sqrt{12}}{\sqrt{3}}$	π^2	$3^{-1} + 3^{-2}$
$7^{-\frac{1}{2}}$	$\frac{22}{7}$	$\sqrt{2} + 1$	$\sqrt{2{\cdot}25}$

2. (a) Write down any rational number between 4 and 6.
(b) Write down any irrational number between 4 and 6.
(c) Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.
(d) Write down any rational number between π and $\sqrt{10}$.

3. (a) For each triangle use Pythagoras' theorem to calculate the length x.
(b) For each triangle state whether the *perimeter* is rational or irrational.
(c) For each triangle state whether the *area* is rational or irrational.
(d) In which triangle is $\sin\theta$ an irrational number?

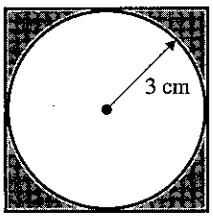


Triangle A



Triangle B

4. The diagram shows a circle of radius 3 cm drawn inside a square. Write down the exact value of the following and state whether the answer is rational or not:
(a) the circumference of the circle
(b) the diameter of the circle
(c) the area of the square
(d) the area of the circle
(e) the shaded area.



5. Think of two *irrational* numbers x and y such that $\frac{x}{y}$ is a *rational* number.

6. Explain the difference between a rational number and an irrational number.

7. (a) Is it possible to multiply a rational number and an irrational number to give an answer which is rational?
(b) Is it possible to multiply two irrational numbers together to give a rational answer?
(c) If either or both are possible, give an example.

Sequences

Exercise 7

Write down each sequence and find the next two numbers.

1. 2, 6, 10, 14
2. 2, 9, 16, 23
3. 95, 87, 79, 71
4. 13, 8, 3, −2
5. 7, 9, 12, 16
6. 20, 17, 13, 8
7. 1, 2, 4, 7, 11
8. 1, 2, 4, 8
9. 55, 49, 42, 34
10. 10, 8, 5, 1
11. −18, −13, −9, −6
12. 120, 60, 30, 15
13. 27, 9, 3, 1
14. 162, 54, 18, 6
15. 2, 5, 11, 20
16. 1, 4, 20, 120
17. 2, 3, 1, 4, 0
18. 720, 120, 24, 6

We can describe a sequence by finding an expression for the nth term of the sequence.

(a) For the sequence 4, 8, 12, 16, ...
The 10th term is $4 \times 10 = 40$.
The nth term is $4n$.
(b) For the sequence (1×2), (2×3), (3×4), (4×5), ...
The 10th term is 10×11.
The nth term is $n(n + 1)$.
(c) For the sequence 5, 7, 9, 11, ...
The common difference between each term is 2 so the expression will contain $2n$.
The nth term is $2n + 3$, by inspection.

Exercise 8

1. Write down each sequence and select the correct formula for the nth term from the list given.

$\boxed{11n}$ $\boxed{10n}$ $\boxed{2n}$ $\boxed{n^2}$ $\boxed{10^n}$ $\boxed{3n}$ $\boxed{100n}$ $\boxed{n^3}$

(a) 2, 4, 6, 8, ...
(b) 10, 20, 30, 40, ...
(c) 3, 6, 9, 12, ...
(d) 11, 22, 33, 44, ...
(e) 100, 200, 300, 400, ...
(f) $1^2, 2^2, 3^2, 4^2, \ldots$
(g) 10, 100, 1000, 10 000, ...
(h) $1^3, 2^3, 3^3, 4^3, \ldots$

2. Look at the sequence: 5, 8, 13, 20, ...
Decide which of the following is the correct expression for the nth term of the sequence.

$\boxed{4n + 1}$ $\boxed{3n + 2}$ $\boxed{n^2 + 4}$

In questions **3** to **10** find a formula for the nth term.

3. 5, 10, 15, 20, ...
4. 2, 4, 8, 16, 32, ...
5. (1×3), (2×4), (3×5), ...
6. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$
7. 7, 14, 21, 28, ...
8. 1, 4, 9, 16, 25, ...
9. $\frac{5}{1^2}, \frac{5}{2^2}, \frac{5}{3^2}, \frac{5}{4^2}, \ldots$
10. $\frac{3}{1}, \frac{4}{2}, \frac{5}{3}, \frac{6}{4}, \ldots$
11. 3, 7, 11, 15, ...
12. 5, 7, 9, 11
13. 7, 5, 3, 1
14. $-5, -1, 3, 7, \ldots$

1.3 Approximations and estimation

Example

(a) 7·8126 = 8 to the nearest whole number
↑ This figure is '5 or more'.
(b) 7·8126 = 7·81 to three significant figures
↑ This figure is not '5 or more'.
(c) 7·8126 = 7·813 to three decimal places
↑ This figure is '5 or more'.
(d) 0·078 126 = 0·0781 to three significant figures.
↑ 7 is the first significant figure.
(e) 3596 = 3600 to two significant figures.
↑ This figure is '5 or more'.

Exercise 9

Write the following numbers correct to:
(a) the nearest whole number (b) three significant figures (c) two decimal places

1. 8·174	**2.** 19·617	**3.** 20·041	**4.** 0·814 52	**5.** 311·14
6. 0·275	**7.** 0·007 47	**8.** 15·62	**9.** 900·12	**10.** 3·555
11. 5·454	**12.** 20·961	**13.** 0·0851	**14.** 0·5151	**15.** 3·071

Write the following numbers correct to one decimal place.

16. 5·71	**17.** 0·7614	**18.** 11·241	**19.** 0·0614	**20.** 0·0081	**21.** 11·12

Measurements and bounds

Measurement is approximate

Example 1

A length of some cloth is measured for a dress. You might say the length is 145 cm to the nearest cm.

The actual length could be anything from 144·5 cm to 145·49999 . . . cm using the normal convention which is to round up a figure of 5 or more. Clearly 145·49999 . . . is effectively 145·5 and we say the *upper bound* is 145·5.

The *lower bound* is 144·5.

As an inequality we can write $144{\cdot}5 \leqslant \text{length} < 145{\cdot}5$

The upper limit often causes confusion. We use 145·5 as the upper bound simply because it is *inconvenient* to work with 145·49999 . . .

Example 2

When measuring the length of a page in a book, you might say the length is 437 mm to the nearest mm.

In this case the actual length could be anywhere from 436·5 mm to 437·5 mm. We write 'length is between 436·5 mm and 437·5 mm'.

In both Examples 1 and 2, the measurement expressed to a given unit is in *possible error* of *half* a *unit*.

Example 3

(a) If you say your weight is 57 kg to the nearest kg, you could actually weigh anything from 56·5 kg to 57·5 kg.

The 'unit' is 1 so 'half a unit' is 0·5

(b) If your brother was weighed on more sensitive scales and the result was 57·2 kg, his actual weight could be from 57·15 kg to 57·25 kg.

The 'unit' is 0·1 so 'half a unit' is 0·05

(c) The weight of a butterfly might be given as 0·032 g. The actual weight could be from 0·0315 g to 0·0325 g.

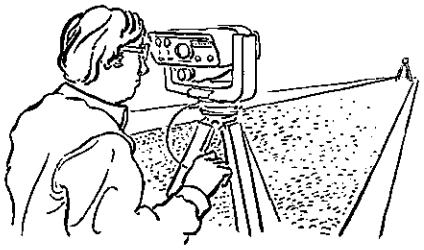
The 'unit' is 0·001 so 'half a unit' is 0·005

Here are some further examples:

Measurement	Lower bound	Upper bound
The diameter of a CD is 12 cm to the nearest cm.	11·5 cm	12·5 cm
The mass of a coin is 6·2 g to the nearest 0·1 g.	6·15 g	6·25 g
The length of a fence is 330 m to the nearest 10 m.	325 m	335 m

Exercise 10

1. In a DIY store the height of a door is given as 195 cm to the nearest cm. Write down the upper bound for the height of the door.
2. A vet weighs a sick goat at 37 kg to the nearest kg. What is the least possible weight of the goat?
3. A cook's weighing scales weigh to the nearest 0·1 kg. What is the upper bound for the weight of a chicken which she weighs at 3·2 kg?
4. A surveyor using a laser beam device can measure distances to the nearest 0·1 m. What is the least possible length of a warehouse which he measures at 95·6 m?
5. In the county sports Jill was timed at 28·6 s for the 200 m. What is the upper bound for the time she could have taken?



6. Copy and complete the table.

	Measurement	Lower bound	Upper bound
(a)	temperature in a fridge = 2 °C to the nearest degree		
(b)	mass of an acorn = 2·3 g to 1 d.p.		
(c)	length of telephone cable = 64 m to nearest m		
(d)	time taken to run 100 m = 13·6 s to nearest 0·1 s		

7. The length of a telephone is measured as 193 mm, to the nearest mm. The length lies between:

A	B	C
192 and 194 mm	192·5 and 193·5 mm	188 and 198 mm

8. The weight of a labrador is 35 kg, to the nearest kg.
The weight lies between:

A	B	C
30 and 40 kg	34 and 36 kg	34·5 and 35·5 kg

9. Liz and Julie each measure a different worm and they both say that their worm is 11 cm long to the nearest cm.
(a) Does this mean that both worms are the same length?
(b) If not, what is the maximum possible difference in the length of the two worms?

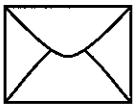
10. To the nearest cm, the length l of a stapler is 12 cm. As an inequality we can write $11{\cdot}5 \leqslant l < 12{\cdot}5$.

For parts (a) to (j) you are given a measurement. Write the possible values using an inequality as above.

(a) mass = 17 kg	(2 s.f.)	(b) $d = 256$ km	(3 s.f.)
(c) length = 2·4 m	(1 d.p.)	(d) $m = 0{\cdot}34$ grams	(2 s.f.)
(e) $v = 2{\cdot}04$ m/s	(2 d.p.)	(f) $x = 12{\cdot}0$ cm	(1 d.p.)
(g) $T = 81{\cdot}4$ °C	(1 d.p.)	(h) $M = 0{\cdot}3$ kg	(1 s.f.)
(i) mass = 0·7 tonnes	(1 s.f.)	(j) $n = 52\,000$	(nearest thousand)

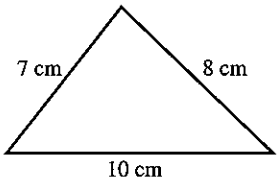
11. A card measuring 11·5 cm long (to the nearest 0·1 cm) is to be posted in an envelope which is 12 cm long (to the nearest cm). Can you guarantee that the card will fit inside the envelope? Explain your answer.


11.5 cm

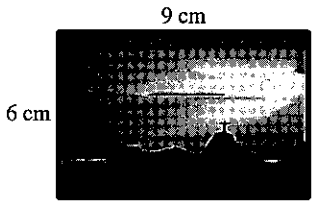

12 cm

Exercise 11

1. The sides of the triangle are measured correct to the nearest cm.
(a) Write down the upper bounds for the lengths of the three sides.
(b) Work out the maximum possible perimeter of the triangle.



2. The dimensions of a photo are measured correct to the nearest cm. Work out the minimum possible area of the photo.



3. In this question the value of a is either exactly 4 or 5, and the value of b is either exactly 1 or 2. Work out:

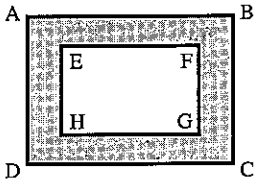
(a) the maximum value of $a + b$ (b) the minimum value of $a + b$
(c) the maximum value of ab (d) the maximum value of $a - b$
(e) the minimum value of $a - b$ (f) the maximum value of $\frac{a}{b}$
(g) the minimum value of $\frac{a}{b}$ (h) the maximum value of $a^2 - b^2$.

4. If $p = 7\,\text{cm}$ and $q = 5\,\text{cm}$, both to the nearest cm, find:
   (a) the largest possible value of $p + q$
   (b) the smallest possible value of $p + q$
   (c) the largest possible value of $p - q$
   (d) the largest possible value of $\frac{p^2}{q}$.

5. If $a = 3{\cdot}1$ and $b = 7{\cdot}3$, correct to 1 decimal place, find the largest possible value of:
   (i) $a + b$ (ii) $b - a$

6. If $x = 5$ and $y = 7$ to one significant figure, find the largest and smallest possible values of:
   (i) $x + y$ (ii) $y - x$ (iii) $\frac{x}{y}$

7. In the diagram, ABCD and EFGH are rectangles with AB = 10 cm, BC = 7 cm, EF = 7 cm and FG = 4 cm, all figures accurate to the nearest cm.
   Find the largest possible value of the shaded area.



8. When a voltage V is applied to a resistance R the power consumed P is given by $P = \frac{V^2}{R}$.
   If you measure V as 12·2 and R as 2·6, correct to 1 d.p., calculate the smallest possible value of P.

Estimation

You should check that the answer to a calculation is 'about the right size'.

Example

Estimate the value of $\frac{57{\cdot}2 \times 110}{2{\cdot}146 \times 46{\cdot}9}$, correct to one significant figure.

We have approximately, $\frac{\cancel{50} \times 100}{2 \times \cancel{50}} \approx 50$

On a calculator the value is 62·52 (to 4 significant figures).

Exercise 12

In this exercise there are 25 questions, each followed by three possible answers. Decide (by estimating) which answer is correct.

1. $7{\cdot}2 \times 9{\cdot}8$	[52·16, 98·36, 70·56]
2. $2{\cdot}03 \times 58{\cdot}6$	[118·958, 87·848, 141·116]
3. $23{\cdot}4 \times 19{\cdot}3$	[213·32, 301·52, 451·62]
4. $313 \times 107{\cdot}6$	[3642·8, 4281·8, 33 678·8]
5. $6{\cdot}3 \times 0{\cdot}098$	[0·6174, 0·0622, 5·98]
6. $1200 \times 0{\cdot}89$	[722, 1068, 131]

7. 0·21 × 93	[41·23, 9·03, 19·53]
8. 88·8 × 213	[18 914·4, 1693·4, 1965·4]
9. 0·04 × 968	[38·72, 18·52, 95·12]
10. 0·11 × 0·089	[0·1069, 0·0959, 0·009 79]
11. 13·92 ÷ 5·8	[0·52, 4·2, 2·4]
12. 105·6 ÷ 9·6	[8·9, 11, 15]
13. 8405 ÷ 205	[4·6, 402, 41]
14. 881·1 ÷ 99	[4·5, 8·9, 88]
15. 4·183 ÷ 0·89	[4·7, 48, 51]
16. 6·72 ÷ 0·12	[6·32, 21·2, 56]
17. 20·301 ÷ 1010	[0·0201, 0·211, 0·0021]
18. 0·288 96 ÷ 0·0096	[312, 102·1, 30·1]
19. 0·143 ÷ 0·11	[2·3, 1·3, 11·4]
20. 159·65 ÷ 515	[0·11, 3·61, 0·31]
21. (5·6 − 0·21) × 39	[389·21, 210·21, 20·51]
22. $\frac{17{\cdot}5 \times 42}{2{\cdot}5}$	[294, 504, 86]
23. (906 + 4·1) × 0·31	[473·21, 282·131, 29·561]
24. $\frac{543 + 472}{18{\cdot}1 + 10{\cdot}9}$	[65, 35, 85]
25. $\frac{112{\cdot}2 \times 75{\cdot}9}{6{\cdot}9 \times 5{\cdot}1}$	[242, 20·4, 25·2]

1.4 Standard form

When dealing with either very large or very small numbers, it is not convenient to write them out in full in the normal way. It is better to use standard form. Most calculators represent large and small numbers in this way.

The number $a \times 10^n$ is in standard form when $1 \leqslant a < 10$ and n is a positive or negative integer.

Example

Write the following numbers in standard form:

(a) $2000 = 2 \times 1000 = 2 \times 10^3$

(b) $150 = 1{\cdot}5 \times 100 = 1{\cdot}5 \times 10^2$

(c) $0{\cdot}000\,4 = 4 \times \frac{1}{10\,000} = 4 \times 10^{-4}$

Note
Most calculators show a display of 2^{03}. You must write 2×10^3.

Exercise 13

Write the following numbers in standard form:

1. 4000	**2.** 500	**3.** 70 000	**4.** 60	**5.** 2400	**6.** 380
7. 46 000	**8.** 46	**9.** 900 000	**10.** 2560	**11.** 0·007	**12.** 0·0004
13. 0·0035	**14.** 0·421	**15.** 0·000 055	**16.** 0·01	**17.** 564 000	**18.** 19 million

19. The population of China is estimated at 1100 000 000. Write this in standard form.

20. A hydrogen atom weighs 0.000 000 000 000 000 000 000 001 67 grams. Write this weight in standard form.

21. The area of the surface of the Earth is about 510 000 000 km^2. Express this in standard form.

22. An atom is 0.000 000 000 25 cm in diameter. Write this in standard form.

23. Avogadro's number is 602 300 000 000 000 000 000 000. Express this in standard form.

24. The speed of light is 300 000 km/s. Express this speed in cm/s in standard form.

Hint
1 km = 1000 m
1 m = 100 cm

25. A very rich oil sheikh leaves his fortune of $\$3{\cdot}6 \times 10^8$ to be divided between his 100 children. How much does each child receive? Give the answer in standard form.

Example

Work out $1500 \times 8\,000\,000$

$$\begin{aligned} 1500 \times 8\,000\,000 &= (1{\cdot}5 \times 10^3) \times (8 \times 10^6) \\ &= 12 \times 10^9 \\ &= 1{\cdot}2 \times 10^{10} \end{aligned}$$

Notice that we multiply the numbers and the powers of 10 separately.

Exercise 14

In questions **1** to **12** give the answer in standard form.

1. 5000×3000	**2.** $60\,000 \times 5000$	**3.** $0{\cdot}000\,07 \times 400$	**4.** $0{\cdot}0007 \times 0{\cdot}000\,01$
5. $8000 \div 0{\cdot}004$	**6.** $(0{\cdot}002)^2$	**7.** $150 \times 0{\cdot}0006$	**8.** $0{\cdot}000\,033 \div 500$
9. $0{\cdot}007 \div 20\,000$	**10.** $(0{\cdot}0001)^4$	**11.** $(2000)^3$	**12.** $0{\cdot}005\,92 \div 8000$

13. If $a = 512 \times 10^2$ $b = 0{\cdot}478 \times 10^6$ $c = 0{\cdot}0049 \times 10^7$
arrange a, b and c in order of size (smallest first).

14. If the number $2{\cdot}74 \times 10^{15}$ is written out in full, how many zeros follow the 4?

15. If the number $7{\cdot}31 \times 10^{-17}$ is written out in full, how many zeros would there be between the decimal point and the first significant figure?

16. If $x = 2 \times 10^5$ and $y = 3 \times 10^{-3}$ correct to one significant figure, find the greatest and least possible values of:

(i) xy (ii) $\dfrac{x}{y}$

Remember
The limits of accuracy of 2 to one significant figure are 1·5 to 2·5

17. Oil flows through a pipe at a rate of $40\,m^3/s$. How long will it take to fill a tank of volume $1{\cdot}2 \times 10^5\,m^3$?

18. Given that $L = 2\sqrt{\frac{a}{k}}$, find the value of L in standard form when $a = 4{\cdot}5 \times 10^{12}$ and $k = 5 \times 10^7$.

19. (a) The number 10 to the power 100 (10 000 sexdecillion) is called a 'Googol'! If it takes $\frac{1}{5}$ second to write a zero and $\frac{1}{10}$ second to write a 'one', how long would it take to write the number 100 'Googols' in full?
    (b) The number 10 to the power of a 'Googol' is called a 'Googolplex'. Using the same speed of writing, how long in years would it take to write 1 'Googolplex' in full? You may assume that your pen has enough ink.

1.5 Ratio and proportion

The word 'ratio' is used to describe a fraction. If the *ratio* of a boy's height to his father's height is 4 : 5, then he is $\frac{4}{5}$ as tall as his father.

Example 1

Change the ratio 2 : 5 into the form

(a) $1 : n$ (b) $m : 1$

(a) $2 : 5 = 1 : \frac{5}{2}$
$= 1 : 2{\cdot}5$

(b) $2 : 5 = \frac{2}{5} : 1$
$= 0{\cdot}4 : 1$

Example 2

Divide \$60 between two people A and B in the ratio 5 : 7.

Consider \$60 as 12 equal parts (i.e. 5 + 7). Then A receives 5 parts and B receives 7 parts.

$\therefore$ A receives $\frac{5}{12}$ of \$60 = \$25

B receives $\frac{7}{12}$ of \$60 = \$35

Example 3

Divide 200 kg in the ratio 1 : 3 : 4.

The parts are $\frac{1}{8}$, $\frac{3}{8}$ and $\frac{4}{8}$ (of 200 kg). i.e. 25 kg, 75 kg and 100 kg.

Exercise 15

In questions **1** to **8** express the ratios in the form $1 : n$.

1. 2 : 6	**2.** 5 : 30	**3.** 2 : 100	**4.** 5 : 8
5. 4 : 3	**6.** 8 : 3	**7.** 22 : 550	**8.** 45 : 360

In questions **9** to **12** express the ratios in the form $n : 1$.

9. 12 : 5	**10.** 5 : 2	**11.** 4 : 5	**12.** 2 : 100

In questions **13** to **18** divide the quantity in the ratio given.

13. \$40; (3 : 5) **14.** \$120; (3 : 7) **15.** 250 m; (14 : 11)
16. \$117; (2 : 3 : 8) **17.** 180 kg; (1 : 5 : 6) **18.** 184 minutes; (2 : 3 : 3)

19. When \$143 is divided in the ratio 2 : 4 : 5, what is the difference between the largest share and the smallest share?

20. Divide 180 kg in the ratio 1 : 2 : 3 : 4.

21. Divide \$4000 in the ratio 2 : 5 : 5 : 8.

22. If $\frac{5}{8}$ of the children in a school are boys, what is the ratio of boys to girls?

23. A man and a woman share a bingo prize of \$1000 between them in the ratio 1 : 4. The woman shares her part between herself, her mother and her daughter in the ratio 2 : 1 : 1. How much does her daughter receive?

24. A man and his wife share a sum of money in the ratio 3 : 2. If the sum of money is doubled, in what ratio should they divide it so that the man still receives the same amount?

25. In a herd of x cattle, the ratio of the number of bulls to cows is 1 : 6. Find the number of bulls in the herd in terms of x.

26. If $x : 3 = 12 : x$, calculate the positive value of x.

27. If $y : 18 = 8 : y$, calculate the positive value of y.

28. \$400 is divided between Ann, Beyoncé and Carol so that Ann has twice as much as Beyoncé and Beyoncé has three times as much as Carol. How much does Beyoncé receive?

29. A cake weighing 550 g has three ingredients: flour, sugar and raisins. There is twice as much flour as sugar and one and a half times as much sugar as raisins. How much flour is there?

30. A brother and sister share out their collection of 5000 stamps in the ratio 5 : 3 The brother then shares his stamps with two friends in the ratio 3 : 1 : 1, keeping most for himself. How many stamps do each of his friends receive?

Proportion

The majority of problems where proportion is involved are usually solved by finding the value of a unit quantity.

Example 1

If a wire of length 2 metres costs \$10, find the cost of a wire of length 35 cm.

200 cm costs 1000 cents

$\therefore$ 1 cm costs $\frac{1000}{200}$ cents = 5 cents

$\therefore$ 35 cm costs 5×35 cents = 175 cents
= \$1·75

Example 2

Eight men can dig a trench in 4 hours. How long will it take five men to dig the same size trench?

8 men take 4 hours
1 man would take 32 hours
5 men would take $\frac{32}{5}$ hours = 6 hours 24 minutes.

Exercise 16

1. Five cans of beer cost \$1·20. Find the cost of seven cans.
2. A man earns \$140 in a 5-day week. What is his pay for 3 days?
3. Three men build a wall in 10 days. How long would it take five men?
4. Nine milk bottles contain $4\frac{1}{2}$ litres of milk between them. How much do five bottles hold?
5. A car uses 10 litres of petrol in 75 km. How far will it go on 8 litres?
6. A wire 11 cm long has a mass of 187 g. What is the mass of 7 cm of this wire?
7. A shopkeeper can buy 36 toys for £20·52. What will he pay for 120 toys?
8. A ship has sufficient food to supply 600 passengers for 3 weeks. How long would the food last for 800 people?
9. The cost of a phone call lasting 3 minutes 30 seconds was 52·5 cents. At this rate, what was the cost of a call lasting 5 minutes 20 seconds?
10. 80 machines can produce 4800 identical pens in 5 hours. At this rate
    (a) how many pens would one machine produce in one hour?
    (b) how many pens would 25 machines produce in 7 hours?
11. Three men can build a wall in 10 hours. How many men would be needed to build the wall in $7\frac{1}{2}$ hours?
12. If it takes 6 men 4 days to dig a hole 3 feet deep, how long will it take 10 men to dig a hole 7 feet deep?
13. Find the cost of 1 km of pipe at 7 cents for every 40 cm.
14. A wheel turns through 90 revolutions per minute. How many degrees does it turn through in 1 second?
15. Find the cost of 20 grams of lead at \$60 per kilogram.

Remember
1 kg = 1000 g

16. The height of an office building is 623 feet. Express this height to the nearest metre using 1 m = 3·281 feet.
17. A floor is covered by 800 tiles measuring 10 cm square. How many square tiles of side 8 cm would be needed to cover the same floor?

18. A battery has enough energy to operate eight toy bears for 21 hours. For how long could the battery operate 15 toy bears?

19. An engine has enough fuel to operate at full power for 20 minutes. For how long could the engine operate at 35% of full power?

20. A large drum, when full, contains 260 kg of oil of density 0·9 g/cm^3. What weight of petrol, of density 0·84 g/cm^3, can be contained in the drum?

21. A wall can be built by 6 men working 8 hours per day in 5 days. How many days will it take 4 men to build the wall if they work only 5 hours per day?

Foreign exchange

Money is changed from one currency into another using the method of proportion.

Exchange rate for US dollars ($):

Country	*Rate of exchange*
Argentina (pesos)	1·01 ARPO = $1
Kuwait (dinar)	0·30 KD = $1
S Arabia (rial)	3·75 SR = $1
UK (pounds)	£0·63 = $1
Euro (euros)	€0·93 = $1

Example

Convert: (a) $22.50 to dinars (b) €300 to dollars.

(a) $1 = 0·30 dinars (KD)

so $22·50 = 0·30 × 22·50 KD

= 6·75 KD

(b) €0·93 = $1

so $€1 = \frac{1}{0·93}$

so $€300 = \$\frac{1}{0·93} \times 300$

= $322.58

Exercise 17

Give your answers correct to two decimal places. Use the exchange rates given in the table.

1. Change the amount of dollars into the foreign currency stated.

(a) $20 [euros] (b) $70 [pounds] (c) $200 [ARPO]

(d) $1·50 [euros] (e) $2·30 [rials] (f) 90c [dinars]

2. Change the amount of foreign currency into dollars.

(a) €500 (b) £2500 (c) €7·5

(d) 900 dinars (e) 125·24 ARPO (f) 750 SR

3. A CD costs £9·50 in Britain and $9·70 in the United States. How much cheaper, in British money, is the CD when bought in the USA?

4. A bottle of Cointreau costs €20·46 in Spain and £12·60 in the UK. Which is the cheaper in dollars, and by how much?

5. The EEC 'Butter Mountain' was estimated in 2004 to be costing €32 860 per day to maintain the storage facilities. How much is this in US dollars?

6. A Jaguar XJS is sold in several countries at the prices given below.

Britain	£15 000
France	€29 490
USA	$25 882

Write out in order a list of the prices converted into pounds.

7. An Irish gentleman on holiday in Germany finds that his wallet contains $700. If he changes the money at a bank how many euros will he receive?

Map scales

You can use proportion to work out map scales. First you need to know these metric equivalents:

1 km = 1000 m	km means kilometre
1 m = 100 cm	m means metre
1 cm = 10 mm	cm means centimetre
	mm means millimetre

Example

A map is drawn to a scale of 1 to 50 000. Calculate:

(a) the length of a road which appears as 3 cm long on the map.
(b) the length on the map of a lake which is 10 km long.

(a) 1 cm on the map is equivalent to 50 000 cm on the Earth.

$\therefore$ 1 cm $\equiv$ 50 000 cm
$\therefore$ 1 cm $\equiv$ 500 m
$\therefore$ 1 cm $\equiv$ 0·5 km

so 3 cm $\equiv 3 \times 0{\cdot}5$ km $= 1{\cdot}5$ km.
The road is 1·5 km long.

(b) 0·5 km $\equiv$ 1 cm
$\therefore$ 1 km $\equiv$ 2 cm
$\therefore$ 10 km $\equiv 2 \times 10$ cm
$= 20$ cm

The lake appears 20 cm long on the map.

Exercise 18

1. Find the actual length represented on a drawing by
   (a) 14 cm (b) 3·2 cm
   (c) 0·71 cm (d) 21·7 cm
   when the scale is 1 cm to 5 m.

2. Find the length on a drawing that represents
   (a) 50 m (b) 35 m
   (c) 7·2 m (d) 28·6 m
   when the scale is 1 cm to 10 m.

3. If the scale is 1 : 10 000, what length will 45 cm on the map represent:
   (a) in cm; (b) in m; (c) in km?

4. On a map of scale 1 : 100 000, the distance between Tower Bridge and Hammersmith Bridge is 12·3 cm. What is the actual distance in km?

5. On a map of scale 1 : 15 000, the distance between Buckingham Palace and Brixton Underground Station is 31·4 cm. What is the actual distance in km?

6. If the scale of a map is 1 : 10 000, what will be the length on this map of a road which is 5 km long?

7. The distance from Hertford to St Albans is 32 km. How far apart will they be on a map of scale 1 : 50 000?

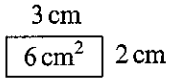
8. The 17th hole at the famous St Andrews golf course is 420 m in length. How long will it appear on a plan of the course of scale 1 : 8000?

An area involves two dimensions multiplied together and hence the scale is multiplied *twice*.

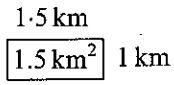
For example, if the linear scale is $\frac{1}{100}$, then the area scale is $\frac{1}{100} \times \frac{1}{100} = \frac{1}{10\,000}$.

You can use a diagram to help:
If a scale is 1 : 50 000
then 2 cm ≡ 1 km

An area of 6 cm^2 can be thought of as:



so the equivalent area using the scale is:



Exercise 19

1. The scale of a map is 1 : 1000. What are the actual dimensions of a rectangle which appears as 4 cm by 3 cm on the map? What is the area on the map in cm^2? What is the actual area in m^2?

2. The scale of a map is 1 : 100. What area does 1 cm^2 on the map represent? What area does 6 cm^2 represent?

3. The scale of a map is 1 : 20 000. What area does 8 cm^2 represent?

4. The scale of a map is 1 : 1000. What is the area, in cm^2, on the map of a lake of area 5000 m^2?

5. The scale of a map is 1 cm to 5 km. A farm is represented by a rectangle measuring 1·5 cm by 4 cm. What is the actual area of the farm?

6. On a map of scale 1 cm to 250 m the area of a car park is 3 cm^2. What is the actual area of the car park in hectares? (1 hectare = 10 000 m^2).

7. The area of the playing surface at Wembley Stadium is $\frac{3}{5}$ of a hectare. What area will it occupy on a plan drawn to a scale of 1 : 500?

8. On a map of scale 1 : 20 000 the area of a forest is 50 cm^2. On another map the area of the forest is 8 cm^2. Find the scale of the second map.

1.6 Percentages

Percentages are simply a convenient way of expressing fractions or decimals. '50% of \$60' means $\frac{50}{100}$ of \$60, or more simply $\frac{1}{2}$ of \$60. Percentages are used very frequently in everyday life and are misunderstood by a large number of people. What are the implications if 'inflation falls from 10% to 8%'? Does this mean prices will fall?

Example

(a) Change 80% to a fraction.
(b) Change $\frac{3}{8}$ to a percentage.
(c) Change 8% to a decimal.

(a) $80\% = \dfrac{80}{100} = \dfrac{4}{5}$

(b) $\dfrac{3}{8} = \left(\dfrac{3}{8} \times \dfrac{100}{1}\right)\% = 37\frac{1}{2}\%$

(c) $8\% = \dfrac{8}{100} = 0{\cdot}08$

Exercise 20

1. Change to fractions:
   (a) 60% (b) 24% (c) 35% (d) 2%

2. Change to percentages:
   (a) $\frac{1}{4}$ (b) $\frac{1}{10}$ (c) $\frac{7}{8}$
   (d) $\frac{1}{3}$ (e) 0·72 (f) 0·31

3. Change to decimals:
   (a) 36% (b) 28% (c) 7%
   (d) 13·4% (e) $\frac{3}{5}$ (f) $\frac{7}{8}$

4. Arrange in order of size (smallest first):
   (a) $\frac{1}{2}$; 45%; 0·6 (b) 0·38; $\frac{6}{16}$; 4%
   (c) 0·111; 11%; $\frac{1}{9}$ (d) 32%; 0·3; $\frac{1}{3}$

5. The following are marks obtained in various tests. Convert them to percentages.
   (a) 17 out of 20 (b) 31 out of 40 (c) 19 out of 80
   (d) 112 out of 200 (e) $2\frac{1}{2}$ out of 25 (f) $7\frac{1}{2}$ out of 20

Example 1

A car costing \$400 is reduced in price by 10%. Find the new price.

$$10\% \text{ of } \$2400 = \frac{10}{100} \times \frac{2400}{1}$$
$$= \$240$$

New price of car $= \$(2400 - 240)$
$= \$2160$

Example 2

After a price increase of 10% a television set costs \$286.
What was the price before the increase?

The price before the increase is 100%.

$\therefore$ 110% of old price $= \$286$

$\therefore$ 1% of old price $= \$\frac{286}{110}$

$\therefore$ 100% of old price $= \$\frac{286}{110} \times \frac{100}{1}$

Old price of TV $= \$260$

Exercise 21

1. Calculate:
   (a) 30% of \$50 (b) 45% of 2000 kg
   (c) 4% of \$70 (d) 2·5% of 5000 people

2. In a sale, a jacket costing \$40 is reduced by 20%. What is the sale price?

3. The charge for a telephone call costing 12 cents is increased by 10%. What is the new charge?

4. In peeling potatoes 4% of the mass of the potatoes is lost as 'peel'. How much is *left* for use from a bag containing 55 kg?

5. Work, to the nearest cent:
   (a) 6·4% of \$15·95 (b) 11·2% of \$192·66
   (c) 8·6% of \$25·84 (d) 2·9% of \$18·18

6. Find the total bill:
   5 golf clubs at \$18·65 each
   60 golf balls at \$16·50 per dozen
   1 bag at \$35·80
   Sales tax at 15% is added to the total cost.

7. In 2000 a club has 250 members who each pay \$95 annual subscription. In 2001 the membership increases by 4% and the annual subscription is increased by 6%. What is the total income from subscriptions in 2001?

8. In 1999 the prison population was 48 700 men and 1600 women. What percentage of the total prison population were men?

9. In 1999 there were 21 280 000 licensed vehicles on the road. Of these, 16 486 000 were private cars. What percentage of the licensed vehicles were private cars?

10. A quarterly telephone bill consists of \$19·15 rental plus 4·7 cents for each dialled unit. Sales tax is added at 15%. What is the total bill for Mrs Jones who used 915 dialled units?

11. Hassan thinks his goldfish got chickenpox. He lost 70% of his collection of goldfish. If he has 60 survivors, how many did he have originally?

12. The average attendance at Parma football club fell by 7% in 1999. If 2030 fewer people went to matches in 1999, how many went in 1998?

13. When heated an iron bar expands by 0·2%. If the increase in length is 1 cm, what is the original length of the bar?

14. In the last two weeks of a sale, prices are reduced first by 30% and then by a *further* 40% of the new price. What is the final sale price of a shirt which originally cost \$15?·

15. During a Grand Prix car race, the tyres on a car are reduced in weight by 3%. If they weigh 388 kg at the end of the race, how much did they weigh at the start?

16. Over a period of 6 months, a colony of rabbits increases in number by 25% and then by a further 30%. If there were originally 200 rabbits in the colony how many were there at the end?

17. A television costs \$270·25 including 15% sales tax. How much of the cost is tax?

18. The cash price for a car was \$7640. Mr Khan bought the car on the following hire purchase terms: 'A deposit of 20% of the cash price and 36 monthly payments of \$191·60'.
Calculate the total amount Mr Khan paid.

Percentage increase or decrease

In the next exercise use the formulae:

$$\text{Percentage profit} = \frac{\text{Actual profit}}{\text{Original price}} \times \frac{100}{1}$$

$$\text{Percentage loss} = \frac{\text{Actual loss}}{\text{Original price}} \times \frac{100}{1}$$

Example 1

A radio is bought for \$16 and sold for \$20. What is the percentage profit?

$$\text{Actual profit} = \$4$$

$$\therefore \quad \text{Percentage profit} = \frac{4}{16} \times \frac{100}{1} = 25\%$$

The radio is sold at a 25% profit.

Example 2

A car is sold for \$2280, at a loss of 5% on the cost price.
Find the cost price.

Do *not* calculate 5% of £2280!

The loss is 5% of the cost price.

$$\therefore \quad 95\% \text{ of cost price} = \$2280$$

$$1\% \text{ of cost price} = \$\frac{2280}{95}$$

$$\therefore \quad 100\% \text{ of cost price} = \$\frac{2280}{95} \times \frac{100}{1}$$

$$\text{Cost price} = \$2400$$

Exercise 22

Hint
¢ is the symbol for cents.
100¢ = $1

1. The first figure is the cost price and the second figure is the selling price. Calculate the percentage profit or loss in each case.
   (a) \$20, \$25 (b) \$400, \$500 (c) \$60, \$54
   (d) \$9000, \$10 800 (e) \$460, \$598 (f) \$512, \$550·40
   (g) \$45, \$39·60 (h) 50¢, 23¢
2. A car dealer buys a car for \$500, gives it a clean, and then sells it for \$640. What is the percentage profit?
3. A damaged carpet which cost \$180 when new, is sold for \$100. What is the percentage loss?
4. During the first four weeks of her life, a baby girl increases her weight from 3·2 kg to 4·7 kg. What percentage increase does this represent? (Give your answer to 3 sig. fig.)
5. When sales tax is added to the cost of a car tyre, its price increases from \$16·50 to \$18·48. What is the rate at which sales tax is charged?
6. In order to increase sales, the price of a Concorde airliner is reduced from £30 000 000 to £28 400 000. What percentage reduction is this?
7. Find the *cost* price of the following:
   (a) selling price \$55, profit 10% (b) selling price \$558, profit 24%
   (c) selling price \$680, loss 15% (d) selling price \$11·78, loss 5%
8. An oven is sold for \$600, thereby making a profit of 20%, on the cost price. What was the cost price?
9. A pair of jeans is sold for \$15, thereby making a profit of 25% on the cost price. What was the cost price?
10. A book is sold for \$5·40, at a profit of 8% on the cost price. What was the cost price?
11. A can of worms is sold for 48¢, incurring a loss of 20%. What was the cost price?
12. A car, which failed its safety test, was sold for \$1430, thereby making a loss of 35% on the cost price. What was the cost price?
13. If an employer reduces the working week from 40 hours to 35 hours, with no loss of weekly pay, calculate the percentage increase in the hourly rate of pay.
14. The rental for a television set changed from \$80 per year to \$8 per month. What is the percentage increase in the yearly rental?
15. A greengrocer sells a melon at a profit of $37\frac{1}{2}\%$ on the price he pays for it. What is the ratio of the cost price to the selling price?
16. Given that $G = ab$, find the percentage increase in G when both a and b increase by 10%.
17. Given that $T = \dfrac{kx}{y}$, find the percentage increase in T when k, x and y all increase by 20%.

Simple interest

When a sum of money $\$P$ is invested for T years at $R\%$ interest per annum (each year), then the interest gained I is given by:

$$I = \frac{P \times R \times T}{100}$$

This is known as simple interest.

Example

Joel invests \$400 for 6 months at 5%.
Work out the simple interest gained.

$P = \$400 \quad R = 5 \quad T = 0{\cdot}5 \quad$ (6 months is half a year)

so $I = \dfrac{400 \times 5 \times 0{\cdot}5}{100}$

$I = \$10$

Exercise 23

1. Calculate:
   (a) the simple interest on \$1200 for 3 years at 6% per annum
   (b) the simple interest on \$700 at 8·25% per annum for 2 years
   (c) the length of time for \$5000 to earn \$1000 if invested at 10% per annum
   (d) the length of time for \$400 to earn \$160 if invested at 8% per annum.
2. Khalid invests \$6750 at 8·5% per annum. How much interest has he earned and what is the total amount in his account after 4 years?
3. Shareen invests \$10 800. After 4 years she has earned \$3240 in interest. At what annual rate of interest did she invest her money?

Compound interest

Suppose a bank pays a fixed interest of 10% on money in deposit accounts. A man puts £500 in the bank.

£ is the symbol for UK pounds sterling.

After one year he has
$500 + 10\%$ of $500 = £550$

After two years he has
$550 + 10\%$ of $550 = £605$

[Check that this is $1{\cdot}10^2 \times 500$]

After three years he has
$605 + 10\%$ of $605 = £665{\cdot}50$

[Check that this is $1{\cdot}10^3 \times 500$]

In general after n years the money in the bank will be $£(1{\cdot}10^n \times 500)$

Exercise 24

1. A bank pays interest of 9% on money in deposit accounts. Mrs Wells puts £2000 in the bank. How much has she after (a) one year, (b) two years, (c) three years?

2. A bank pays interest of 11%. Mr Olsen puts £5000 in the bank. How much has he after (a) one year, (b) three years, (c) five years?

3. A computer operator is paid £10 000 a year. Assuming her pay is increased by 7% each year, what will her salary be in four years time?

4. Mrs Bergkamp's salary in 2001 is £30 000 per year. Every year her salary is increased by 5%.
   In 2002 her salary will be $30\,000 \times 1{\cdot}05$ = £31 500
   In 2003 her salary will be $30\,000 \times 1{\cdot}05 \times 1{\cdot}05$ = £33 075
   In 2004 her salary will be $30\,000 \times 1{\cdot}05 \times 1{\cdot}05 \times 1{\cdot}05$ = £34 728·75
   And so on.
   (a) What will her salary be in 2005?
   (b) What will her salary be in 2007?

5. The price of a house was £90 000 in 1998. At the end of each year the price is increased by 6%.
   (a) Find the price of the house after 1 year.
   (b) Find the price of the house after 3 years.
   (c) Find the price of the house after 10 years.

6. Assuming an average inflation rate of 8%, work out the probable cost of the following items in 10 years:
   (a) car £6500
   (b) T.V. £340
   (c) house £50 000

7. A new car is valued at £15 000. At the end of each year its value is reduced by 15% of its value at the start of the year. What will it be worth after 3 years?

8. The population of an island increases by 10% each year.
   After how many years will the original population be doubled?

9. A bank pays interest of 11% on £6000 in a deposit account.
   After how many years will the money have trebled?

10. A tree grows in height by 21% per year. It is 2 m tall after one year.
    After how many more years will the tree be over 20 m tall?

11. Which is the better investment over ten years:
    £20 000 at 12% compound interest
    or £30 000 at 8% compound interest?

1.7 Speed, distance and time

Calculations involving these three quantities are simpler when the speed is *constant*. The formulae connecting the quantities are as follows:

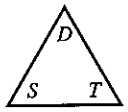
(a) distance = speed × time

(b) $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

(c) $\text{time} = \dfrac{\text{distance}}{\text{speed}}$

A helpful way of remembering these formulae is to write the letters D, S and T in a triangle,

thus:



to find D, cover D and we have ST

to find S, cover S and we have $\dfrac{D}{T}$

to find T, cover T and we have $\dfrac{D}{S}$

Great care must be taken with the units in these questions.

Example 1

A man is running at a speed of 8 km/h for a distance of 5200 metres. Find the time taken in minutes.

5200 metres = 5·2 km

$$\text{time taken in hours} = \left(\frac{D}{S}\right) = \frac{5{\cdot}2}{8}$$

$$= 0{\cdot}65 \text{ hours}$$

time taken in minutes = 0·65 × 60

= 39 minutes

Example 2

Change the units of a speed of 54 km/h into metres per second.

54 km/hour = 54 000 metres/hour

$$= \frac{54\,000}{60} \text{ metres/minute}$$

$$= \frac{54\,000}{60 \times 60} \text{ metres/second}$$

$$= 15 \text{ m/s}$$

Exercise 25

1. Find the time taken for the following journeys:
   (a) 100 km at a speed of 40 km/h
   (b) 250 miles at a speed of 80 miles per hour
   (c) 15 metres at a speed of 20 cm/s (answer in seconds)
   (d) 10^4 metres at a speed of 2·5 km/h

2. Change the units of the following speeds as indicated:
   (a) 72 km/h into m/s
   (b) 108 km/h into m/s
   (c) 300 km/h into m/s
   (d) 30 m/s into km/h
   (e) 22 m/s into km/h
   (f) 0·012 m/s into cm/s
   (g) 9000 cm/s into m/s
   (h) 600 miles/day into miles per hour
   (i) 2592 miles/day into miles per second

3. Find the speeds of the bodies which move as follows:
   (a) a distance of 600 km in 8 hours
   (b) a distance of 31·64 km in 7 hours
   (c) a distance of 136·8 m in 18 seconds
   (d) a distance of 4×10^4 m in 10^{-2} seconds
   (e) a distance of 5×10^5 cm in 2×10^{-3} seconds
   (f) a distance of 10^8 mm in 30 minutes (in km/h)
   (g) a distance of 500 m in 10 minutes (in km/h)

4. Find the distance travelled (in metres) in the following:
   (a) at a speed of 55 km/h for 2 hours
   (b) at a speed of 40 km/h for $\frac{1}{4}$ hour
   (c) at a speed of 338·4 km/h for 10 minutes
   (d) at a speed of 15 m/s for 5 minutes
   (e) at a speed of 14 m/s for 1 hour
   (f) at a speed of 4×10^3 m/s for 2×10^{-2} seconds
   (g) at a speed of 8×10^5 cm/s for 2 minutes

5. A car travels 60 km at 30 km/h and then a further 180 km at 160 km/h. Find:
   (a) the total time taken
   (b) the average speed for the whole journey.

6. A cyclist travels 25 kilometres at 20 km/h and then a further 80 kilometres at 25 km/h. Find:
   (a) the total time taken
   (b) the average speed for the whole journey.

7. A swallow flies at a speed of 50 km/h for 3 hours and then at a speed of 40 km/h for a further 2 hours. Find the average speed for the whole journey.

8. A runner ran two laps around a 400 m track. She completed the first lap in 50 seconds and then decreased her speed by 5% for the second lap. Find:
   (a) her speed on the first lap
   (b) her speed on the second lap
   (c) her total time for the two laps
   (d) her average speed for the two laps.
9. The airliner Concorde flies 2000 km at a speed of 1600 km/h and then returns due to bad weather at a speed of 1000 km/h. Find the average speed for the whole trip.
10. A train travels from A to B, a distance of 100 km, at a speed of 20 km/h. If it had gone two and a half times as fast, how much earlier would it have arrived at B?
11. Two men running towards each other at 4 m/s and 6 m/s respectively are one kilometre apart. How long will it take before they meet?
12. A car travelling at 90 km/h is 500 m behind another car travelling at 70 km/h in the same direction. How long will it take the first car to catch the second?
13. How long is a train which passes a signal in twenty seconds at a speed of 108 km/h?
14. A train of length 180 m approaches a tunnel of length 620 m. How long will it take the train to pass completely through the tunnel at a speed of 54 km/h?
15. An earthworm of length 15 cm is crawling along at 2 cm/s. An ant overtakes the worm in 5 seconds. How fast is the ant walking?
16. A train of length 100 m is moving at a speed of 50 km/h. A horse is running alongside the train at a speed of 56 km/h. How long will it take the horse to overtake the train?
17. A car completes a journey at an average speed of 40 km/h. At what speed must it travel on the return journey if the average speed for the complete journey (out and back) is 60 km/h?

Mixed problems

Exercise 26

1. Fill in the blank spaces in the table so that each row contains equivalent values.

fraction	decimal	percentage
	0·28	
		64%
$\frac{5}{8}$		

2. An engine pulls four identical carriages. The engine is $\frac{2}{3}$ the length of a carriage and the total length of the train is 86·8 m. Find the length of the engine.

3. A wedding cake is made from the ingredients listed below.

   500 g flour, 450 g butter, 470 g sugar,
   1·8 kg mixed fruit, 4 eggs (weighing 70 g each)

   The cake loses 12% of its weight during cooking. What is its final weight?

4. Abdul left his home at 7.35 a.m. and drove at an average speed of 45 km/h arriving at the airport at 8.50 a.m. How far is his home from the airport?

5. Joe's parents have agreed to lend him 60% of the cost of buying a second-hand car. If Joe still has to find $328 himself, how much does the car cost?

6. Which bag of potatoes is the better value:
   Bag A, 6 kg for $4·14 or
   Bag B, 2·5 kg for $1·80?

7. An aeroplane was due to take off from Madrid airport at 18:42 but it was 35 min late. During the flight, thanks to a tail wind, the plane made up the time and in fact landed 16 min before its scheduled arrival time of 00:05. (Assume that the plane did not cross any time zones on its journey.)
   (a) What time did the aeroplane take off?
   (b) What time did it land?

8. A 20 cent coin is 1·2 mm thick. What is the value of a pile of 20 cent coins which is 21·6 cm high?

9. Work out $\frac{3}{5} + 0{\cdot}12 + 6\%$ of 10.

Exercise 27

1. Find the distance travelled by light in one hour, given that the speed of light is 300 000 kilometres per second.
   Give the answer in kilometres in standard form.

2. When the lid is left off an ink bottle, the ink evaporates at a rate of $2{\cdot}5 \times 10^{-6}$ cm^3/s. A full bottle contains 36 cm^3 of ink. How long, to the nearest day, will it take for all the ink to evaporate?

3. Convert 3·35 hours into hours and minutes.

Remember
There are 60 minutes in 1 hour.

4. When I think of a number, multiply it by 6 and subtract 120, my answer is −18. What was my original number?

5. The cost of advertising in a local paper for one week is:

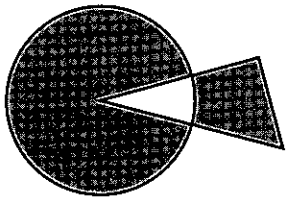
28 cents per word plus 75 cents

(a) What is the cost of an advertisement of 15 words for one week?
(b) What is the greatest number of words in an advertisement costing up to \$8 for one week?
(c) If an advertisement is run for two weeks, the cost for the second week is reduced by 30%. Calculate the total cost for an advertisement of 22 words for two weeks.

6. Bronze is made up of zinc, tin and copper in the ratio 1 : 4 : 95. A bronze statue contains 120 g of tin. Find the quantities of the other two metals required and the total weight of the statue.

Exercise 28

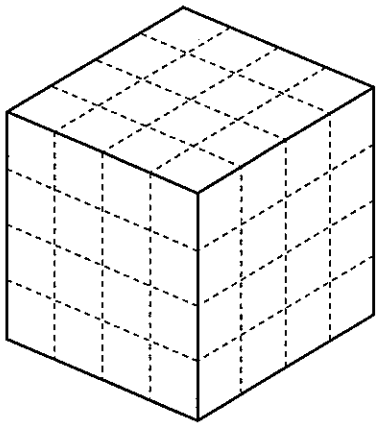
1. In the diagram $\frac{5}{6}$ of the circle is shaded and $\frac{2}{3}$ of the triangle is shaded. What is the ratio of the area of the circle to the area of the triangle?



2. Find the exact answer to the following by first working out a rough answer and then using the information given.
Do *not* use a calculator.
(a) If $142{\cdot}3 \times 98{\cdot}5 = 14\,016{\cdot}55$ find $140{\cdot}1655 \div 14{\cdot}23$
(b) If $76{\cdot}2 \times 8{\cdot}6 = 655{\cdot}32$ find $6553{\cdot}2 \div 86$
(c) If $22{\cdot}3512 \div 0{\cdot}268 = 83{\cdot}4$ find $8340 \times 26{\cdot}8$
(d) If $1{\cdot}6781 \div 17{\cdot}3 = 0{\cdot}097$ find $9700 \times 0{\cdot}173$

3. A sales manager reports an increase of 28% in sales this year compared to last year.
The increase was \$70 560.
What were the sales last year?

4. Small cubes of side 1 cm are stuck together to form a large cube of side 4 cm. Opposite faces of the large cube are painted the same colour, but adjacent faces are different colours. The three colours used are red, black and green.
(a) How many small cubes have just one red and one green face?
(b) How many small cubes are painted on one face only?
(c) How many small cubes have one red, one green and one black face?
(d) How many small cubes have no faces painted?



5. The bullet from a rifle travels at a speed of 3×10^4 cm/s. Work out the length of time in seconds taken for the bullet to hit a target 54 m away.
6. A sewing machine cost \$162·40 after a price increase of 16%. Find the price before the increase.
7. To get the next number in a sequence you double the previous number and subtract two.
   The fifth number in the sequence is 50.
   Find the first number.
8. A code uses 1 for A, 2 for B, 3 for C and so on up to 26 for Z. Coded words are written without spaces to confuse the enemy, so 18 could be AH or R. Decode the following message.

   208919 919 1 2251825 199121225 31545
9. A coach can take 47 passengers. How many coaches are needed to transport 1330 passengers?

1.8 Calculator

In this book, the keys are described thus:

[+]	add	[√]	square root
[−]	subtract	[x^2]	square
[×]	multiply	[$1/x$]	reciprocal
[÷]	divide	[y^x]	raise number y to the power x
[=]	equals		

Using the [ANS] button

The [ANS] button can be used as a 'short term memory'.
It holds the answer from the previous calculation.

Example

Evaluate the following to 4 significant figures:

(a) $\dfrac{5}{1{\cdot}2 - 0{\cdot}761}$ (b) $\left(\dfrac{1}{0{\cdot}084}\right)^4$ (c) $\sqrt[3]{[3{\cdot}2 \times (1{\cdot}7 - 1{\cdot}64)]}$

(a) Find the bottom line first.

[1·2] [−] [0·761] [EXE] [5] [÷] [ANS] [EXE]

The calculator reads 11·38952164

$\therefore$ Answer = 11·39 (to four sig. fig.)

Note: The [EXE] button works the same as the [=] button.

(b) $\left(\frac{1}{0{\cdot}084}\right)^4$

$\boxed{0{\cdot}084}$ $\boxed{1/x}$ $\boxed{y^x}$ $\boxed{4}$ $\boxed{=}$

Answer 20 090 (to four sig. fig.)

(c) $\sqrt[3]{[3{\cdot}2(1{\cdot}7 - 1{\cdot}64)]}$

$\boxed{1{\cdot}7}$ $\boxed{-}$ $\boxed{1{\cdot}64}$ $\boxed{=}$ $\boxed{\times}$ $\boxed{3{\cdot}2}$ $\boxed{=}$

$\boxed{y^x}$ $\boxed{0{\cdot}333\,333}$ $\boxed{=}$

Answer 0·5769 (to four sig. fig.)

Note: To find a cube root, raise to the power $\frac{1}{3}$, or as a decimal 0·333 ...

Exercise 29

Use a calculator to evaluate the following, giving the answers to 4 significant figures:

1. $\frac{7{\cdot}351 \times 0{\cdot}764}{1{\cdot}847}$
2. $\frac{0{\cdot}0741 \times 14\,700}{0{\cdot}746}$
3. $\frac{0{\cdot}0741 \times 9{\cdot}61}{23{\cdot}1}$
4. $\frac{417{\cdot}8 \times 0{\cdot}008\,41}{0{\cdot}073\,24}$
5. $\frac{8{\cdot}41}{7{\cdot}601 \times 0{\cdot}008\,47}$
6. $\frac{4{\cdot}22}{1{\cdot}701 \times 5{\cdot}2}$
7. $\frac{9{\cdot}61}{17{\cdot}4 \times 1{\cdot}51}$
8. $\frac{8{\cdot}71 \times 3{\cdot}62}{0{\cdot}84}$
9. $\frac{0{\cdot}76}{0{\cdot}412 - 0{\cdot}317}$
10. $\frac{81{\cdot}4}{72{\cdot}6 + 51{\cdot}92}$
11. $\frac{111}{27{\cdot}4 + 2960}$
12. $\frac{27{\cdot}4 + 11{\cdot}61}{5{\cdot}9 - 4{\cdot}763}$
13. $\frac{6{\cdot}51 - 0{\cdot}1114}{7{\cdot}24 + 1{\cdot}653}$
14. $\frac{5{\cdot}71 + 6{\cdot}093}{9{\cdot}05 - 5{\cdot}77}$
15. $\frac{0{\cdot}943 - 0{\cdot}788}{1{\cdot}4 - 0{\cdot}766}$
16. $\frac{2{\cdot}6}{1{\cdot}7} + \frac{1{\cdot}9}{3{\cdot}7}$
17. $\frac{8{\cdot}06}{5{\cdot}91} - \frac{1{\cdot}594}{1{\cdot}62}$
18. $\frac{4{\cdot}7}{11{\cdot}4 - 3{\cdot}61} + \frac{1{\cdot}6}{9{\cdot}7}$
19. $\frac{3{\cdot}74}{1{\cdot}6 \times 2{\cdot}89} - \frac{1}{0{\cdot}741}$
20. $\frac{1}{7{\cdot}2} - \frac{1}{14{\cdot}6}$
21. $\frac{1}{0{\cdot}961} \times \frac{1}{0{\cdot}412}$
22. $\frac{1}{7} + \frac{1}{13} - \frac{1}{8}$
23. $4{\cdot}2\left(\frac{1}{5{\cdot}5} - \frac{1}{7{\cdot}6}\right)$
24. $\sqrt{(9{\cdot}61 + 0{\cdot}1412)}$
25. $\sqrt{\left(\frac{8{\cdot}007}{1{\cdot}61}\right)}$
26. $(1{\cdot}74 + 9{\cdot}611)^2$
27. $\left(\frac{1{\cdot}63}{1{\cdot}7 - 0{\cdot}911}\right)^2$
28. $\left(\frac{9{\cdot}6}{2{\cdot}4} - \frac{1{\cdot}5}{0{\cdot}74}\right)^2$
29. $\sqrt{\left(\frac{4{\cdot}2 \times 1{\cdot}611}{9{\cdot}83 \times 1{\cdot}74}\right)}$
30. $(0{\cdot}741)^3$
31. $(1{\cdot}562)^5$
32. $(0{\cdot}32)^3 + (0{\cdot}511)^4$
33. $(1{\cdot}71 - 0{\cdot}863)^6$
34. $\left(\frac{1}{0{\cdot}971}\right)^4$
35. $\sqrt[3]{(4{\cdot}714)}$
36. $\sqrt[3]{(0{\cdot}9316)}$
37. $\sqrt[3]{\left(\frac{4{\cdot}114}{7{\cdot}93}\right)}$
38. $\sqrt[4]{(0{\cdot}8145 - 0{\cdot}799)}$
39. $\sqrt[5]{(8{\cdot}6 \times 9{\cdot}71)}$
40. $\sqrt[3]{\left(\frac{1{\cdot}91}{4{\cdot}2 - 3{\cdot}766}\right)}$
41. $\left(\frac{1}{7{\cdot}6} - \frac{1}{18{\cdot}5}\right)^3$
42. $\frac{\sqrt{(4{\cdot}79)} + 1{\cdot}6}{9{\cdot}63}$
43. $\frac{(0{\cdot}761)^2 - \sqrt{(4{\cdot}22)}}{1{\cdot}96}$
44. $\sqrt[3]{\left(\frac{1{\cdot}74 \times 0{\cdot}761}{0{\cdot}0896}\right)}$
45. $\left(\frac{8{\cdot}6 \times 1{\cdot}71}{0{\cdot}43}\right)^3$
46. $\frac{9{\cdot}61 - \sqrt{(9{\cdot}61)}}{9{\cdot}61^2}$
47. $\frac{9{\cdot}6 \times 10^4 \times 3{\cdot}75 \times 10^7}{8{\cdot}88 \times 10^6}$
48. $\frac{8{\cdot}06 \times 10^{-4}}{1{\cdot}71 \times 10^{-6}}$

49. $\dfrac{3{\cdot}92 \times 10^{-7}}{1{\cdot}884 \times 10^{-11}}$ **50.** $\left(\dfrac{1{\cdot}31 \times 2{\cdot}71 \times 10^{5}}{1{\cdot}91 \times 10^{4}}\right)^{5}$ **51.** $\left(\dfrac{1}{9{\cdot}6} - \dfrac{1}{9{\cdot}99}\right)^{10}$ **52.** $\dfrac{\sqrt[3]{(86{\cdot}6)}}{\sqrt[4]{(4{\cdot}71)}}$

53. $\dfrac{23{\cdot}7 \times 0{\cdot}0042}{12{\cdot}48 - 9{\cdot}7}$ **54.** $\dfrac{0{\cdot}482 + 1{\cdot}6}{0{\cdot}024 \times 1{\cdot}83}$ **55.** $\dfrac{8{\cdot}52 - 1{\cdot}004}{0{\cdot}004 - 0{\cdot}0083}$ **56.** $\dfrac{1{\cdot}6 - 0{\cdot}476}{2{\cdot}398 \times 41{\cdot}2}$

57. $\left(\dfrac{2{\cdot}3}{0{\cdot}791}\right)^{7}$ **58.** $\left(\dfrac{8{\cdot}4}{28{\cdot}7 - 0{\cdot}47}\right)^{3}$ **59.** $\left(\dfrac{5{\cdot}114}{7{\cdot}332}\right)^{5}$ **60.** $\left(\dfrac{4{\cdot}2}{2{\cdot}3} + \dfrac{8{\cdot}2}{0{\cdot}52}\right)^{3}$

61. $\dfrac{1}{8{\cdot}2^{2}} - \dfrac{3}{19^{2}}$ **62.** $\dfrac{100}{11^{3}} + \dfrac{100}{12^{3}}$ **63.** $\dfrac{7{\cdot}3 - 4{\cdot}291}{2{\cdot}6^{2}}$ **64.** $\dfrac{9{\cdot}001 - 8{\cdot}97}{0{\cdot}95^{3}}$

65. $\dfrac{10{\cdot}1^{2} + 9{\cdot}4^{2}}{9{\cdot}8}$ **66.** $(3{\cdot}6 \times 10^{-8})^{2}$ **67.** $(8{\cdot}24 \times 10^{4})^{3}$

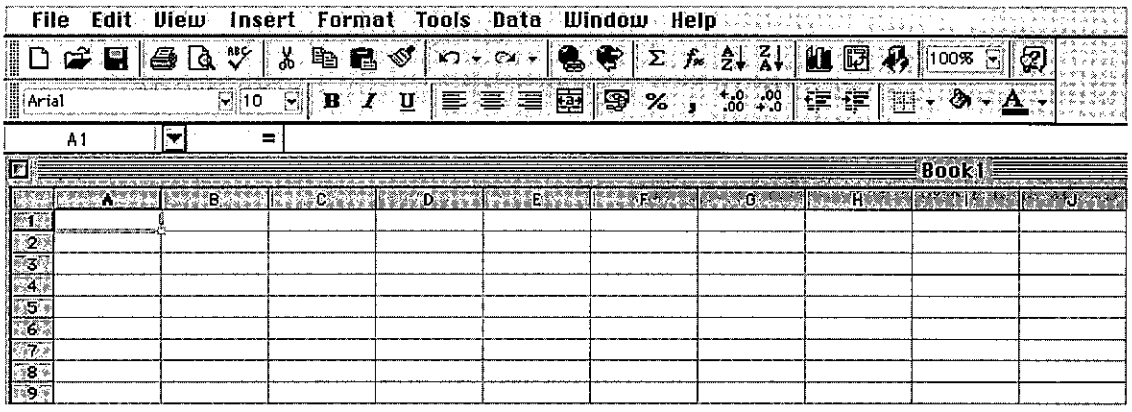
68. $(2{\cdot}17 \times 10^{-3})^{3}$ **69.** $(7{\cdot}095 \times 10^{-6})^{\frac{1}{3}}$ **70.** $\sqrt[3]{\left(\dfrac{4{\cdot}7}{2{\cdot}3^{2}}\right)}$

1.9 Using a spreadsheet on a computer

This section is written for use with Microsoft Excel. Other spreadsheet programs work in a similar way.

Select Microsoft Excel from the desk top.

A spreadsheet appears on your screen as a grid with rows numbered 1, 2, 3, 4, ... and the columns lettered A, B, C, D, ...
The result should be a window like the one below.



Cell The spaces on the spreadsheet are called cells. Individual cells are referred to as A1, B3, F9, like grid references. Cells may contain *labels*, *values* or *formulae*. The current cell has a black border.

Label Any words, headings or messages used to help the layout and organisation of the spreadsheet.

Value A number placed in a cell. It may be used as input to a calculation.

Tasks 1, 2 and 3 are written for you to become familiar with how the main functions of a spreadsheet program work. Afterwards there are sections on different topics where spreadsheets can be used.

Task 1. To generate the whole numbers from 1 to 10 in column A.

(a) In cell A1 type '1' and press *Return*. This will automatically take you to the cell below. [NOTE that you must use the *Return* button and not the arrow keys to move down the column.]
(b) In cell A2 type the formula '= A1 + 1' and press *Return*. [NOTE that the = sign is needed before any formula.]
(c) We now want to copy the formula in A2 down column A as far as A10. Click on A2 again and put the arrow in the bottom right corner of cell A2 (a + sign will appear) and drag down to A10.

Task 2. To generate the odd numbers in column B.

(a) In B1 type '1' (press *Return*).
(b) In B2 type the formula ' = B1 + 2' (press *Return*).
(c) Click in B2 and copy the formula down column B as far as B10.

Task 3. To generate the first 15 square numbers.

(a) As before generate the numbers from 1 to 15 in cells A1 to A15.
(b) In B1 put the formula ' = A1 * A1' and press *Return*.
(c) Click in B1 and copy the formula down as far as B15.

Pie charts and bar charts using a spreadsheet on a computer

Example

Display the data about the activities in one day.

Enter the headings: *Sleep* in A1, *School* in B1 etc. [Use the *tab* key to move across the page.]

Enter the data: 8 in A2, 7 in B2 etc.

Book1

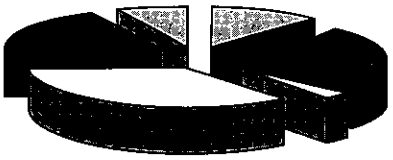
	A	B	C	D	E	F	G	H
1	Sleep	School	TV	Eating	Homework	Other		
2	8	7	1.5	1	1.5	5		
3								
4								
5								
6								
7								

Now highlight all the cells from A1 to F2. [Click on A1 and drag across to F2.]

Click on the () Chart wizard on the toolbar.

Select 'pie' and then choose one of the examples displayed. Follow the on-screen prompts.

Alternatively, for a bar chart, select 'charts' after clicking on the chart wizard. Proceed as above.



You will be able to display your charts with various '3D' effects possibly in colour. This approach is recommended when you are presenting data that you have collected as part of an investigation.

Scatter graphs on a computer

Example

Plot a scatter graph showing the marks of 10 students in Maths and Science.

Enter the headings: *Maths* in A1, *Science* in B1
Enter the data as shown.

	A	B
1	Maths	Science
2	23	30
3	45	41
4	73	67
5	35	74
6	67	77
7	44	50
8	32	41
9	66	55
10	84	70
11	36	32

Now highlight all the cells from A2 to B11.
[Click on A1 and drag across and down to B11.]

Click on the () Chart wizard on the toolbar.

Select XY (Scatter) and select the picture which looks like a scatter graph.

Follow the on-screen prompts.

On 'Titles' enter:

Chart title:	Maths/Science results
Value (X) axis:	Maths
Value (Y) axis:	Science

Experiment with 'Axes', 'Gridlines', 'Legend' and 'Data Labels'.

Task Enter the data on a spreadsheet and print a scatter graph. What does each scatter graph show?

(a)

Height	Armspan
162	160
155	151
158	157
142	144
146	148
165	163
171	167
148	150
150	147

(b)

Temperature	Sales
23	7
18	14
7	23
20	9
4	30
12	19
15	15
18	15
10	20

Revision exercise 1A

1. Evaluate, without a calculator:
(a) $148 \div 0{\cdot}8$ (b) $0{\cdot}024 \div 0{\cdot}000\,16$
(c) $(0{\cdot}2)^2 \div (0{\cdot}1)^3$ (d) $2 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4}$
(e) $1\frac{3}{4} \times 1\frac{3}{5}$ (f) $\dfrac{1\frac{1}{6}}{1\frac{2}{3} + 1\frac{1}{4}}$

2. On each bounce, a ball rises to $\frac{4}{5}$ of its previous height. To what height will it rise after the third bounce, if dropped from a height of 250 cm?

3. A man spends $\frac{1}{3}$ of his salary on accommodation and $\frac{2}{5}$ of the remainder on food. What fraction is left for other purposes?

4. $a = \frac{1}{2}$, $b = \frac{1}{4}$. Which one of the following has the greatest value?
(i) ab (ii) $a + b$ (iii) $\dfrac{a}{b}$ (iv) $\dfrac{b}{a}$ (v) $(ab)^2$

5. Express 0·054 73:
(a) correct to three significant figures
(b) correct to three decimal places
(c) in standard form.

6. Evaluate $\frac{2}{3} + \frac{4}{7}$, correct to three decimal places.

7. Evaluate the following and give the answer in standard form:
(a) $3600 \div 0{\cdot}000\,12$ (b) $\dfrac{3{\cdot}33 \times 10^4}{9 \times 10^{-1}}$ (c) $(30\,000)^3$

8. (a) \$143 is divided in the ratio 2 : 3 : 6; calculate the smallest share.
(b) A prize is divided between three people X, Y and Z. If the ratio of X's share to Y's share is 3 : 1 and Y's share to Z's share is 2 : 5, calculate the ratio of X's share to Z's share.
(c) If $a : 3 = 12 : a$, calculate the positive value of a.

9. Labour costs, totalling \$47·25, account for 63% of a car repair bill. Calculate the total bill.

10. (a) Convert to percentages:
(i) 0·572 (ii) $\frac{7}{8}$
(b) Express 2·6 kg as a percentage of 6·5 kg.
(c) In selling a red herring for 92c, a fishmonger makes a profit of 15%. Find the cost price of the fish.

11. The length of a rectangle is decreased by 25% and the breadth is increased by 40%. Calculate the percentage change in the area of the rectangle.

12. (a) What sum of money, invested at 9% interest per year, is needed to provide an income of \$45 per year?
(b) A particle increases its speed from 8×10^5 m/s to $1{\cdot}1 \times 10^6$ m/s. What is the percentage increase?

13. An English family on holiday in France exchanged £450 for euros when the exchange rate was 1·41 euros to the pound. They spent 500 euros and then changed the rest back into pounds, by which time the exchange rate had become 1·46 euros to the pound. How much did the holiday cost? (Answer in pounds.)

14. Given that

$$t = 2\pi\sqrt{\left(\frac{l}{g}\right)},$$

find the value of t, to three sig. fig., when $l = 2{\cdot}31$ and $g = 9{\cdot}81$

15. A map is drawn to a scale of 1 : 10 000. Find:
(a) the distance between two railway stations which appear on the map 24 cm apart.
(b) the area, in square kilometres, of a lake which has an area of 100 cm^2 on the map.

16. A map is drawn to a scale of 1 : 2000. Find:
(a) the actual distance between two points, which appear 15 cm apart on the map.
(b) the length on the map of a road, which is 1·2 km in length.
(c) the area on the map of a field, with an actual area of 60 000 m^2.

17. (a) On a map, the distance between two points is 16 cm. Calculate the scale of the map if the actual distance between the points is 8 km.
(b) On another map, two points appear 1·5 cm apart and are in fact 60 km apart. Calculate the scale of the map.

18. (a) A house is bought for \$20 000 and sold for \$24 400. What is the percentage profit?
(b) A piece of meat, initially weighing 2·4 kg, is cooked and subsequently weighs 1·9 kg. What is the percentage loss in weight?
(c) An article is sold at a 6% loss for \$225·60. What was the cost price?

19. (a) Convert into metres per second:
(i) 700 cm/s (ii) 720 km/h (iii) 18 km/h
(b) Convert into kilometres per hour:
(i) 40 m/s (ii) 0·6 m/s

20. (a) Calculate the speed (in metres per second) of a slug which moves a distance of 30 cm in 1 minute.
(b) Calculate the time taken for a bullet to travel 8 km at a speed of 5000 m/s.
(c) Calculate the distance flown, in a time of four hours, by a pigeon which flies at a speed of 12 m/s.

21. A motorist travelled 200 km in five hours. Her average speed for the first 100 km was 50 km/h. What was her average speed for the second 100 kilometres?

22. 1 3 8 9 10

From these numbers, write down:

(a) the prime number, (Note: 1 is NOT a prime number)

(b) a multiple of 5,

(c) two square numbers,

(d) two factors of 32.

(e) Find two numbers m and n from the list such that $m = \sqrt{n}$ and $n = \sqrt{81}$.

(f) If each of the numbers in the list can be used once, find p, q, r, s, t such that $(p + q)r = 2(s + t) = 36$.

23. The value of t is given by

$$t = 2\pi\sqrt{\left(\frac{2{\cdot}31^2 + 0{\cdot}9^2}{2{\cdot}31 \times 9{\cdot}81}\right)}.$$

Without using a calculator, and using suitable approximate values for the numbers in the formula, find an estimate for the value of t. (To earn the marks in this question you must show the various stages of your working.)

24. Throughout his life Mr Cram's heart has beat at an average rate of 72 beats per minute. Mr Cram is sixty years old. How many times has his heart beat during his life? Give the answer in standard form correct to two significant figures.

25. Estimate the answer correct to one significant figure. Do not use a calculator.

(a) $(612 \times 52) \div 49{\cdot}2$

(b) $(11{\cdot}7 + 997{\cdot}1) \times 9{\cdot}2$

(c) $\sqrt{\left(\frac{91{\cdot}3}{10{\cdot}1}\right)}$

(d) $\pi\sqrt{(5{\cdot}2^2 + 18{\cdot}2)}$

26. Evaluate the following using a calculator: (answers to four sig. fig.)

(a) $\dfrac{0{\cdot}74}{0{\cdot}81 \times 1{\cdot}631}$

(b) $\sqrt{\left(\dfrac{9{\cdot}61}{8{\cdot}34 - 7{\cdot}41}\right)}$

(c) $\left(\dfrac{0{\cdot}741}{0{\cdot}8364}\right)^4$

(d) $\dfrac{8{\cdot}4 - 7{\cdot}642}{3{\cdot}333 - 1{\cdot}735}$

27. Evaluate the following and give the answers to three significant figures:

(a) $\sqrt[3]{(9{\cdot}61 \times 0{\cdot}0041)}$

(b) $\left(\dfrac{1}{9{\cdot}5} - \dfrac{1}{11{\cdot}2}\right)^3$

(c) $\dfrac{15{\cdot}6 \times 0{\cdot}714}{0{\cdot}0143 \times 12}$

(d) $\sqrt[4]{\left(\dfrac{1}{5 \times 10^3}\right)}$

28. The edges of a cube are all increased by 10%. What is the percentage increase in the volume?

Examination exercise 1B

1. A family arrives home at 01:10 after a journey that took $7\frac{1}{2}$ hours. At what time on the previous day did their journey start?

N 95 2

2. After adding a profit of 20%, the selling price of a television is \$684. Calculate the cost price. N 95 2

3. Insert one of the symbols $>$, $=$, $<$ to make each of the statements correct.

(a) $(0{\cdot}2)^2$ $\quad 4 \times 10^{-2}$ $\qquad$ (b) $\frac{37}{73}$ $\quad 0{\cdot}507$ $\qquad$ N 95 2

4. A map has a scale of 1 : 50 000.

(a) A road on the map is 10 cm long. What is the real length of the road in kilometres?

(b) The area of a farm on the map is $6\,\text{cm}^2$. What is the real area of the farm in hectares?
[1 hectare $= 10\,000\,\text{m}^2 = 0{\cdot}01\,\text{km}^2$] J 96 2

5. $\frac{82}{99}$, 82%, $\sqrt{0{\cdot}674}$

(a) Write these in order of size, starting with the smallest.

(b) Find the difference between the largest and the smallest, giving your answer correct to two significant figures. J 97 2

6. The ratio of men : women : children living in Newtown is 6 : 7 : 3. There are 42 000 women.

(a) (i) How many children live in Newtown?
(ii) How many people altogether live in Newtown?

(b) The 42 000 women is an increase of 20% on the number of women ten years ago.
Calculate how many women lived in Newtown ten years ago.

(c) Twelve thousand of the children attend school and 48% of them are boys.
(i) Calculate the number of boys and the number of girls at school.
(ii) The average age of the 12 000 children is exactly 10·54 years. The average age of the boys is exactly 10·35 years. Calculate the average age of the girls, correct to two decimal places. J 98 4

7. A cinema has 200 seats. Ticket prices are \$5 for an adult and \$2·50 for a child.

(a) One evening, 80% of the seats in the cinema are occupied. Twenty of the people present are children. Calculate the total money taken from the sale of tickets.

(b) Another evening, x children are present and **all** the seats are occupied. The money taken for tickets is \$905.
(i) Write down an equation in x.
(ii) Calculate the value of x.

You can find out about equations on page 52.

(c) The money taken for tickets for a week is \$10 800. This sum is divided between costs, wages and profit in the ratio 2 : 3 : 7. Calculate:
(i) the profit for the week,
(ii) the simple interest earned if this profit is invested at a rate of 5% per annum for 4 months. J 97 4

8. Ahmed earns \$20 000 each year.

(a) In 1991, he paid no tax on the first \$3000 of his earnings. He paid 25% of the rest as tax. Show that he paid \$4250 as tax.

(b) In 1992, he paid no tax on the first \$4000 of his earnings. He paid 30% of the rest as tax. Calculate how much he paid as tax.

(c) In 1993, he paid no tax on the first \$$x$ of his earnings. He paid 30% of the rest as tax.
(i) Find an expression in terms of x for the amount of tax he paid.
(ii) Calculate the value of x if he paid \$4950 as tax. J 95 4

9. (a) As the product of its prime factors,
$1080 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$.
Write 135, 210 and 1120 as the product of their prime factors.

(b) Copy this grid.

$a = 1$	$b =$	$c =$
$d =$	$e =$	$f =$
$g =$	$h =$	$i = 8$

The nine digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are to be placed in your grid in such a way that the following four statements are *all* true.

$$a \times b \times d \times e = 135$$
$$b \times c \times e \times f = 1080$$
$$d \times e \times g \times h = 210$$
$$e \times f \times h \times i = 1120$$

The digits 1 and 8 have already been placed for you.

Use your answers to part (a) to answer the following questions.

(i) Which is the only digit, other than 1, that is a factor of 135, 1080, 210 *and* 1120?

(ii) Which is the only letter to appear in all four statements above?

(iii) 7 is a factor of only two of the numbers 135, 1080, 210 and 1120. Which two?

(c) Now complete the grid. N 96 4

10. The first five terms of a sequence are 4, 9, 16, 25, 36, ...
Find:
(a) the 10th term,
(b) the nth term. N 03 2

11. $\mathscr{E} = \{-2\frac{1}{2}, -1, \sqrt{2}, 3{\cdot}5, \sqrt{30}, \sqrt{36}\}$
$X = \{\text{integers}\}$
$Y = \{\text{irrational numbers}\}$
List the members of:
(a) X,
(b) Y. N 03 2

12. Abdul invested \$240 when the rate of simple interest was r% per year.
After m **months** the interest was \$$I$.
Write down and simplify an expression for I, in terms of m and r.
N 03 2

13. A baby was born with a mass of 3·6 kg.
After three months this mass had increased to 6 kg.
Calculate the percentage increase in the mass of the baby. N 03 2

14. In 1950, the population of Switzerland was 4 714 900
In 2000, the population was 7 087 000.
(a) Work out the percentage increase in the population from 1950 to 2000.
(b) (i) Write the 1950 population correct to 3 significant figures.
(ii) Write the 2000 population in standard form. J 03 2